

**MATHS METHODS 3 & 4**

**TRIAL** **EXAMINATION 2**

**SOLUTIONS**

**2019**

# SECTION A – Multiple-choice answers

1. **E 9. E 17. D**
2. **A 10. D 18. B**
3. **D 11. D 19. E**
4. **B 12. D 20. A**
5. **E 13. B**
6. **E 14. C**
7. **E 15. C**
8. **C 16. D**

# SECTION A – Multiple-choice solutions

# Question 1



The answer is E.

**Question 2**



Do a quick sketch, noting the restricted domain.

The endpoints are at .

The range is .

The answer is A.



**Question 3**

Sketch the graph of *g.*



The answer is D.

**Question 4**

Since , we know that the point  is translated 1 unit in the negative direction of the *x*-axis to become the point .

It is dilated by a factor of 3 from the *x*-axis to become the point .

It is translated 2 units in the negative direction of the *y*-axis to become the point .

So point *Q* is the point.

The answer is B.

**Question 5**

Method 1



The answer is E.

Method 2



The answer is E.

**Question 6**

Since *X* has a binomial distribution,

****

The answer is E.

**Question 7**



The answer is E.

**Question 8**



Since *A* and *B* are mutually exclusive, .

Draw a Venn diagram.

Let .

Since .

So .

The answer is C.

**Question 9**



The answer is E.

**Question 10**



Point of tangency is .

Equation of tangent is 



*y*-intercept occurs when 



The answer is D.

**Question 11**



The answer is D.

# Question 12

Sketch the graphs using your CAS.

For , there are no asymptotes over the interval .

For , there are two asymptotes over the interval.

For  there is exactly one asymptote.

The answer is D.

**Question 13**



The closest answer is 0.0021.

The answer is B.

**Question 14**

Stationary points occur when .



So stationary points occur when 

There will be two solutions to this quadratic equation in *x* (and therefore two stationary points) when the discriminant is greater than zero.



but *p* is positive (given in question) so .

The answer is C.

**Question 15**



Solve  for *x* using CAS.





The closest answer is 3.6.

The answer is C.

**Question 16**

Both the functions *f* and *g* are symmetrical about the *y*-axis so for example,



The shaded area is given by 

The answer is D.

**Question 17**





The answer is D.

**Question 18**

The graph has been reflected in the *y*-axis and then dilated by a factor of 2 from the *x*-axis or vice-versa. Note that the point  does not change.

Only option B reflects this.

The answer is B.

**Question 19**

Using CAS, solve .





etc.

Since  then  are the two possible values of *k.*

Only  is offered in the answers in option E.

The answer is E.

**Question 20**

The function *f* and *g* are power functions.

Since , we know that *f* and *g* are inverse functions and their graphs are therefore reflections of each other in the line .

We know that regardless of the value of *p*, the graphs will intersect at the points .

An indicative graph is shown below.



The average value of *f* is the height of a rectangle having the same area as the area under the graph of *f* for a given interval.

Looking at the graph above over the interval , the average value of *f* would always be less than the average value of *g* regardless of the value of *p* since  over this interval.

The same can’t be said for the other options.

The answer is A.

**SECTION B**

**Question 1** (12 marks)

1. **i.** Define  on your CAS.



Height of waste awaiting processing is zero at .

There is waste material awaiting processing in the machine for 3 minutes after the container starts being poured.

**(1 mark)**

1.   **(1 mark)**



**(1 mark)**

1. **i.** The solid line shown below represents the total height.



**(1 mark)** for 

**(1 mark)** for 

1. Method 1 – using the graph

The maximum height is 4 metres and it occurs at .

**(1 mark)** correct height

**(1 mark)** correct times

Method 2 – algebraically and graphically

Total height for 



The maximum height is 4 metres and it occurs at .

**(1 mark)** correct height

**(1 mark)** correct times

1. The graph of  is translated *a* units to the right to become the graph of . **(1 mark)**
2. We are told in the question that container 2 is poured after container 1 and since

.

Also, since there is some waste from container 1 left in the machine when container 2 starts being poured then from the graph .

So .

**(1 mark)**

1. From the graph, .

Note that both endpoints of this interval are excluded.

**(1 mark)**

1. When the graphs of  intersect,  as seen on the graph.

So instead of solving  we need only solve .

Solve for *t* using CAS.



From the graph, when , the graphs of  can’t be intersecting ( would be outside its domain ie the second container would have had to start being poured before ).

So when , the graphs are intersecting and the total height of waste would be  metres.

 **(1 mark)**



From the graph below, if  then the total height of waste (i.e. ) would be greater than  metres.



If , the total height of waste is less than or equal to  metres.

So we require 

**(1 mark)**

**Question 2** (11 marks)

1. From the graph, since the curtains are symmetrical, .

**(1 mark)**

1. The graphs of  are symmetrical about the *y*-axis. That is, they are a reflection of each other in the *y*-axis. To obtain the rule for *l*, replace *x* with −*x* in the rule for *r*.



**(1 mark)**

1. total area **(1 mark)**



**(1 mark)**

1. 

Let 

For a dilation by a factor of 2 units from the *y*-axis we replace *x* with .

The rule becomes



**(1 mark)**

For a translation of 1 unit to the left we replace *x* with .

The rule becomes



So the rule for  is



**(1 mark)**

To define a function you must give the rule AND the domain.

Since , the left endpoint remains zero after the dilation but is moved 1 unit left by the translation.

So .

**(1 mark)**

1. Solve  for *x* using CAS.



**(1 mark)**

1. Area of double layer of curtain



**(1 mark)**

Area of no curtain



**(1 mark)**

Area of single layer of curtain



**(1 mark)**

**Question 3** (11 marks)

1. Stationary points occur when .

Define  on your CAS.

 **(1 mark)**

The stationary points are . **(1 mark)**

1. Tangent is  using CAS.

**(1 mark)**

1. Using CAS, solve simultaneously the equations .

The lines intersect at  i.e. the lines intersect at two points only.

**(1 mark)**

1. Do a quick sketch.



Using your CAS,



**(1 mark)**

**(1 mark)**

**(1 mark)**

**(1 mark)**

1. **i.** Using the graph we see that there



are exactly 2 points of intersection

between 

 passes through the maximum

turning point of the graph of

.

This turning point occurs when

. **(1 mark)**

Solving for *x*, .



Note that *a* must be a positive constant.

**(1 mark)**

* 1. When , the equation  has exactly two solutions.



**(1 mark)**

1. From part **e. i.**,  and there are two tangents to the graph of *f* with a gradient of 2.

From part **e. ii.,** one occurs when  and the other when .

The equation of the tangent to the graph of *f* when  from part **b.**.

The other tangent passes through .

The equation of the second tangent is



**(1 mark)**



The two tangents are shown above.

The tangent  passes through the origin.

The *x*-intercept of the tangent .



So this tangent is translated 0.84375 units in the positive direction of the *x*-axis to coincide with the graph of .

So 

**(1 mark)**

**Question 4** (16 marks)

1. Let *F* be the number of cars in the sample with faulty airbags.

 **(1 mark)**

Method 1 – using CAS



**(1 mark)**

(Using CAS )

Method 2 – using the binomial formula



**(1 mark)**

1. Using CAS,  the confidence interval is given by  correct to four decimal places.

Note that the number of successes you have to enter in your CAS is obtained by calculating , ie for the given sample of 20 cars, the proportion of these cars with a faulty airbag was 0.15. This means that 3 out of the 20 cars had a faulty airbag.

**(1 mark)** – correct left endpoint

**(1 mark)** – correct right endpoint

1. From part **a.**  i.e. *F* represents the **number** of cars in a sample which have a faulty airbag.





**(1 mark)**

**(1 mark)**

**(1 mark)**

Note that since the sample size is small, we cannot use a normal approximation.

1. Let *X* represent the time taken by a car owner to respond to the recall.





  **(1 mark)**

1.  (Conditional probability)



**(1 mark)**

**(1 mark)**

1. We have 

Using CAS we have





So (correct to 4 decimal places).

**(1 mark)**

1. Let *m* be the median time, in minutes, taken to replace the airbags.

Solve  for *m*. **(1 mark)**



Note that 

**(1 mark)**

1. Let *Y* represent the time taken, in minutes, to replace the airbags.





 **(1 mark)**

|  |  |  |  |
| --- | --- | --- | --- |
|  | *F* | *M* |  |
| <1 hour | = **0.28** | 0.4052 | **0.6852**  (from part **h.**) |
| ≥1 hour | 0.14 | 0.1748 | 0.3148 |
|  | **0.42** | **0.58** | **1** |

**(1 mark)**

All the values in bold are values that we are given or already have and the rest can be calculated from these. This is a conditional probability question since we know we have a male car owner.



 (correct to 3 decimal places) **(1 mark)**

**Question 5** (10 marks)

1. Define *f* on your CAS.

**(1 mark)**





Since *f* is a positive cubic function (ie the coefficient of the term is a positive number ie 1), the local maximum occurs at  and the local minimum occurs at .

**(1 mark)**

1. average rate of change







So 



So average rate of change = *k* as required.

**(1 mark)**

1. Draw a graph of *f*.



Since the function *h* is strictly decreasing and has a maximal domain, then, using the coordinates of the points found in part **a.**, . **(1 mark)**

Also, .

**(1 mark)**

1. Draw a graph of *h* and .



Because of the symmetry of the graphs of the functions *h* and  about the line ,



**(1 mark)**

**(1 mark)**

1. From part **b.**, we saw that .

The line .

This straight line has an *x*-intercept of .



The graph of  is shown above together with the region required which is shaded.

Since we don't know the rule for , we find an equivalent area which is shaded in the diagram below. This equivalent area is the area enclosed by the graphs of *h*  and

, which we were told earlier in the question passes through the points . Since we **do** know the rule for *h*, we can calculate the area.





**(1 mark)** – correct terminals

**(1 mark)** – correct integrand

**(1 mark)** – correct answer